

Gaussian Random Field: Physical Origin of Sersic Profiles

Renyue Cen¹

ABSTRACT

While the Sersic profile family provide adequate fits for the surface brightness profiles of observed galaxies, the physical origin is unknown. We show that, if the cosmological density field are seeded by random gaussian fluctuations, as in the standard cold dark matter model, galaxies with steep central profiles have simultaneously extended envelopes of shallow profiles in the outskirts, whereas galaxies with shallow central profiles are accompanied by steep density profiles in the outskirts. These properties are in accord with those of the Sersic profile family. Moreover, galaxies with steep central profiles form their central regions in smaller denser subunits that possibly merge subsequently, which naturally leads to formation of bulges. In contrast, galaxies with shallow central profiles form their central regions in a coherent fashion without significant substructure, a necessary condition for disk galaxy formation. Thus, the scenario is self-consistent with respect to the correlation between observed galaxy morphology and Sersic index. We predict further that clusters of galaxies should display a similar trend, which should be verifiable observationally.

1. Introduction

The process of galaxy formation has likely imprinted useful information in the stellar structures. A great amount of effort has been invested in characterizing detailed stellar structures of galaxies of all types, dating back to Plummer (1911) for globular clusters and Reynolds (1913) for the Andromeda, and if one is so inclined, to Kant (1755) who might be the first contemplating the shape of the Milky Way and island universes. In modern times, among the best known examples, the de Vaucouleurs (1948) law - surface brightness $I(R) \propto e^{-kR^{1/4}}$ (where R is radius and k a normalization constant) - describes giant elliptical galaxies well, whereas the King (1962) law appears to provide better fits for fainter elliptical galaxies; disk galaxies are in most cases described by the exponential disk model (Hodge 1971): $I(R) \propto e^{-kR}$. The major advantage of Sersic (1968) profile family - $I(R) \propto e^{-kR^{1/n}}$ - is that they provide an encompassing set of profiles with n from less than 1 to as large as 10, including the exponential disk ($n = 1$) and de Vaucouleurs ($n = 4$) model.

Even at the age of sophisticated hydrodynamic simulations, the physical origin of the Sersic profile family that have well described all galaxies remains enigmatic. This author is of the opinion that the nature of galaxy formation process in the context of modern cosmological structure formation model is perhaps too complex to warrant any possibility of analytic fits to be accurate beyond the zero-th order. While efforts to characterize deviations from or additions to the standard fits are

¹Princeton University Observatory, Princeton, NJ 08544; cen@astro.princeton.edu

not only necessary but also very important to account for rich galaxy data (e.g., Lauer et al. 1995), it would also seem beneficial to construe the basic trend displayed by the wide applicability of the Sersic profile family to enhance our physical understanding of the galaxy formation process.

In this *Letter* we provide a basic physical understanding of the Sersic profile family in the context of the standard cosmological model with gaussian random density field. Our simple analysis provides, for the first time, a self-consistent physical origin for the Sersic profile family. This also opens up the possibilities to explore the physical links to other properties of galaxies, since, for example, it comes natural and apparently inevitable that the steep profiled galaxies have a much higher fraction of substructures that form early and interactions/mergers among them would lead to formation of elliptical galaxies, enabling a self-consistent picture. This study is the fifth paper in the series “On the Origin of the Hubble Sequence”.

2. Gaussian Random Field and Sersic Profiles

The standard cosmological constant dominated cold dark matter cosmological model has a number of distinct features. One of the most important is that the initial density fluctuations are gaussian and random. As a result, the statistical properties are fully determined by a vector quantity, namely, the linear power spectrum of the density fluctuations, P_k , which is well determined by observations from the microwave experiments and others (e.g., Komatsu et al. 2011). Observational evidence is that allowed deviations from gaussianity are at the level of 10^{-3} and less in the linear regime (Planck Collaboration et al. 2013).

In a gaussian random field, different waves are superimposed on one another in a random fashion, with the ensemble of waves at a given length following gaussian distribution and the square of the mean equal to the amplitude of the power spectrum at that wavelength. Here, a simple illustration is shown to contain rich physics and can already account for the basic trend of the Sersic profiles, which, more importantly, are additionally in accord with properties of galaxies other than the profiles.

Figure 1 shows an example of the formation of a massive galaxy that contains small-scale fluctuations with large amplitude (left panel) and an example of the formation of a massive galaxy that contains small-scale fluctuations with small amplitude (right panel). In both panels peaks that are above the horizontal red dot-dashed line would have collapsed by $z = 1$. Our choice of redshift $z = 1$ has no material consequence and we expect the generic trends should not depend on that choice.

In the left panel we see that, between the two points where the blue dashed curve intersect the horizontal red dot-dashed line, there are three separate density peaks with peak amplitude of $6 - 7$. Thus, a significant portion of the three peaks would have collapsed by redshift $z = 4 - 6$ to form three separate galaxies. Note that structures formed at higher redshifts tend to be denser than structures formed at lower redshifts. Therefore, these earlier structures would settle to form the dense central region. Although it is probable that the galaxies formed at the three separate peaks subsequently merge to form a dense elliptical galaxy, our conclusion of forming a dense central region

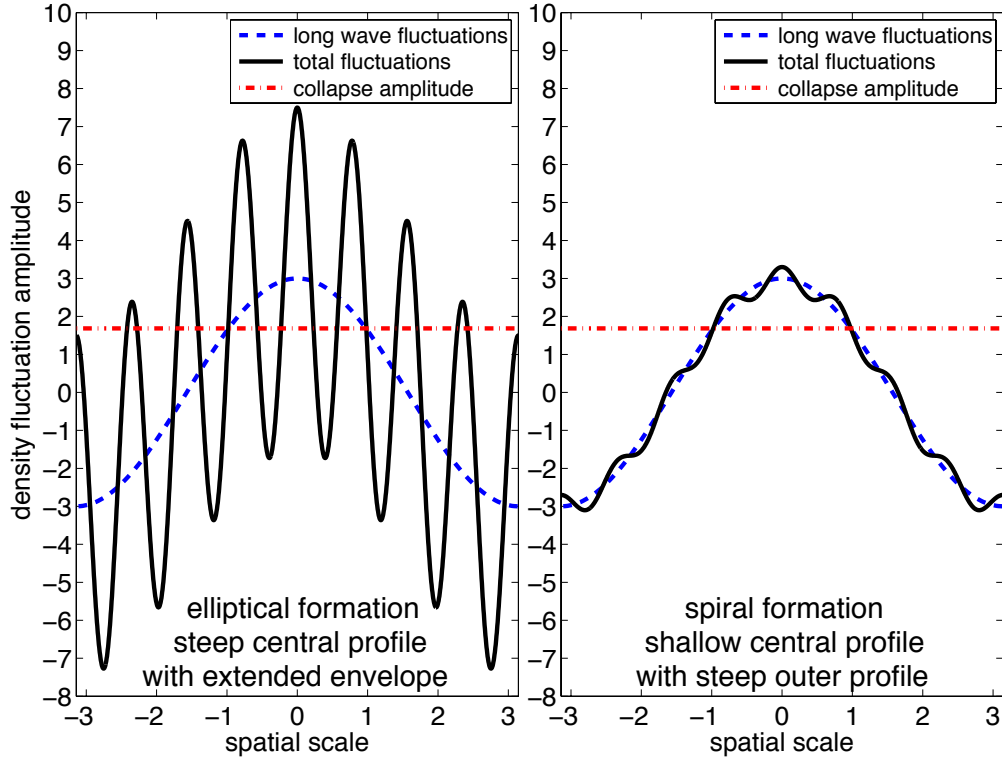


Fig. 1.— **Left panel:** shows an example of the formation of a massive galaxy at $z = 1$ that is overall determined by the large waves indicated by the blue dashed curve. On top of the large linear wave, there is small-scale linear wave of length $1/8$ of the large one with a fluctuation amplitude 1.5 times larger than the large wave. Both long and short waves are chosen to be sinusoidal for the illustration. The sum of the long and short waves is shown as the black solid curve. The horizontal red dot-dashed line of amplitude value 1.68 indicates the amplitude of the fluctuation that has collapsed by $z = 1$. **Right panel:** same as the left panel except the small-scale wave has a fluctuation amplitude 10 times smaller than the large wave. Note that the examples are shown in 1-d but meant to be in 3-d.

in this case does not necessarily require all of them to merge. Moreover, there are two somewhat smaller peaks at x values of ~ -1.5 and $\sim +1.5$ of amplitude ~ 4.5 , which would have collapsed by redshift $z = 2 - 4$. In addition, there two still smaller peaks at x values of ~ -2.5 and $\sim +2.5$ of amplitude ~ 2.5 , which would have collapsed by redshift $z = 1 - 2$. It is reasonable to expect that the four outer small galaxies would accrete onto the central galaxy to form the outer envelope by $z = 0$. Thus, this configuration would form a central dense structure with a steep profile due to the early formation of the central subunits and their subsequent descent to the center (and possible merging), and an extended envelope due to later infall of small galaxies that form in outer regions at some earlier times, resulting in a profile resembling a Sersic profile with $n \gg 1$. This overall picture seems to resemble two-phase formation scenario for elliptical galaxies from detailed cosmological hydrodynamic simulations (Oser et al. 2010; Lackner et al. 2011).

In the right panel we see that, between the two points where the blue dashed curve intersect the horizontal red dot-dashed line, there is no significant substructure. Therefore, the collapse of the central region will be rather coherent without significant central condensation (i.e., without a stellar bulge). Furthermore, there is no significant density peak outside the central region that has collapsed; as a result, there is little stellar envelope due to late infall of small galaxies. Thus, this configuration would form a galaxy with a shallow central density slope and a very steep outer slope. We suggest that this configuration would form a bulge-less spiral galaxy with a profile similar to a Sersic profile with $n = 1$. A corollary is that the configuration depicted in the right panel would occur in a “quiet” environment, which may be quantitatively described as having a small pair-wise velocity dispersion (Davis & Peebles 1983) or a high Mach number (Suto et al. 1992). Our local environment appears to belong to this category. Perhaps this explains why there is preponderance of giant bulge-less galaxies in our neighborhood (Kormendy et al. 2010). This does not necessarily suggest that the observed large fraction ($\sim 50\%$) of large bulge-less galaxies in our local universe is representative for the universe as a whole. Our own expectation is that the fraction of large bulge-less galaxies, averaged over the entire universe, will be substantially lower than that seen in the very local neighborhood. Future surveys with resolutions as good as those for local galaxies now can check this.

It is easy to imagine a variety of configurations that may fall in-between these two (nearly) bookend examples. Since the gaussian density fluctuation is “compensated” in the sense that the large density peak tends to be sandwiched by a pair of troughs, the expected trend is this: a larger degree of central substructure is accompanied by a larger degree of substructure in the outskirts, whereas a lesser degree of central substructure is accompanied by a lesser degree of substructure in the outskirts. Since the total density fluctuations are linear combinations of each independent waves, one can generalize the configurations from two waves to an arbitrary number of waves but the trend seen in Figure 1 remains. In short, the generic trend obtained essentially hinges on two important features of the gaussian random field: each density wave is *compensated* and *independent*.

3. Discussion and Conclusions

Based on a simple analysis we show that gaussian random field provides the physical origin for the observed Sersic profiles. The two unique properties of the gaussian random field - waves are compensated and independent - dictate that a more central concentrated stellar structure of a galaxy is simultaneously accompanied by an extended stellar envelope, and vice versa. Additionally, those with steep inner slopes are expected to contain significant subunits that form early and coalesce later, which are consistent with the paradigm of merger driven formation of elliptical galaxies, whether being dry (van Albada 1982) or wet (Hopkins et al. 2006). On the other hand, those with shallow inner slopes are expected to contain little substructure, which would bode well for the formation of disk galaxies. Thus, the picture is self-consistent.

This analysis is illustrative and qualitative. It will be useful later to formulate a model that is quantitative and statistical in the context of the gaussian field statistics (e.g., Bardeen et al. 1986; Bond et al. 1991). The task in hand is, however, still more complex than a full statistical analysis

of the gaussian random field, because baryonic physics is expected to play an important role. Cosmological reionization (a.k.a, photoheating of the intergalactic medium), gravitational shock heating due to large-scale structure formation and feedback from stellar evolution and supermassive black hole growth may quantitatively change the stellar makeup in both the “central region” and “outer region” shown in Figure 1, perhaps to varying degrees. Nevertheless, we see no physical reason that any of these baryonic processes will alter qualitatively the systematic trend that is illustrated in the previous section.

Since the examples shown in Figure 1 are generic in terms of spatial scales, we expect that our argument is applicable on other scales. If the above analysis on galaxies is extended to clusters of galaxies, the following new predictions are made. Clusters of galaxies are expected to display a similar trend or a family of density profiles from concentrated ones resembling those of elliptical galaxies with large Sersic index n to less concentrated ones resembling those of disk galaxies with small Sersic index n . For this purpose, the characterization of density profiles of clusters of galaxies should be performed with respect to the stellar component. Procedurally, one first identifies the virial radius of the cluster and then finds the best Sersic fit within the virial radius. Two critical issues are that cluster members are properly identified and projection effects minimized, and that intra-cluster light is accounted for. As an example, clusters with cD galaxies should display the shallowest slope and the most extended distribution of galaxies in the outer regions, resulting in a very high n value if fit with Sersic profile for the stellar density. This prediction is verifiable. Although a tight correlation between the presence of cooling flows in X-ray clusters and the presence of cD galaxies at the center is observed, it becomes natural to expect such an outcome from our analysis, because the presence of cD galaxies means early formation of at least the “seed” of the central region that is denser in gas and dark matter as well as in stars.

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